

Quiz No. 3

March 31, 2006

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1} = 1.986 \text{ Btu lbmol}^{-1} \text{ R}^{-1}$$

Do all questions, which are of equal value.

1. Methane is compressed adiabatically in a pipeline pumping station from 100 psia and 60 °F to 400 psia at the rate of 10 lb s⁻¹. The compressor efficiency is 0.80. Estimate the power requirement of the compressor and the discharge temperature of the methane by the following methods:

- (a) Methane is taken to be an ideal gas with

$$C_p/R = 1.702 + 0.009081 (T/K) - 2.164 \times 10^{-6} (T/K)^2$$

- (b) Properties of methane are represented by the attached P-H diagram

2. A binary system consisting of 3 mol of benzene (1) and 2 mol of ethylbenzene (2) at 100 kPa is at its bubblepoint temperature.

- (a) Estimate this temperature and the composition of the incipient phase.
 (b) If a feed consisting of 70 mol of benzene and 30 mol of ethylbenzene is charged into a separator maintained at 100 kPa and the above-calculated temperature, what would be the amount of vapor and that of liquid in the separator?

The vapor pressures of benzene and ethylbenzene are represented by the following equations:

$$\ln P_1^{\text{sat}}/\text{kPa} = 13.7819 - 2726.81/[(t/^\circ\text{C}) + 217.572]$$

$$\ln P_2^{\text{sat}}/\text{kPa} = 13.9726 - 3259.93/[(t/^\circ\text{C}) + 212.300]$$

ln 0.6

e

x₁ = 0.6
5/10

P₁ P₂ (P₁^{sat} - P₁^{sat}) x₁

ChE 223 Chemical Thermodynamics

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1. Methane is compressed adiabatically in a pipeline pumping station from 100 psia and 60 °F to 400 psia at the rate of $10 \text{ lb}_m \text{ s}^{-1}$. The compressor efficiency is 0.80. Estimate the power requirement of the compressor and the discharge temperature of the methane by the following methods:

(a) Methane is taken to be an ideal gas with

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(b) Properties of methane are represented by the attached P-H diagram

Solution:

$$(a) T_1 = 60 \text{ °F} = (60 + 459.67) \text{ R} / (1.8 \text{ R/K}) = 288.7 \text{ K}$$

For an ideal gas

$$\begin{aligned} S &= \int_{T_1}^{T_2} (C_p/T) dT - R \int_{P_1}^{P_2} (1/P) dP \\ &= R \int_{T_1}^{T_2} [(C_p/R)/T] dT - R \int_{P_1}^{P_2} (1/P) dP \\ &= R [A \ln(T_2/T_1) + B(T_2 - T_1) + (C/2)(T_2^2 - T_1^2) - \ln(P_2/P_1)] \end{aligned}$$

For a reversible adiabatic process, $\Delta S = 0$. Thus, for the given coefficient values and the specified values of the process variables, T_2 is found to be 392.3 K*.

*This can be done by means of Newton's method as follows:

$$T^{(i+1)} = T^{(i)} - f/f'$$

where

$$f = A \ln(T^{(i)}/T_1) + B(T^{(i)} - T_1) + (C/2)(T^{(i)2} - T_1^2) - \ln(P_2/P_1)$$

$$f' = df/dT = A/T^{(i)} + B + C T^{(i)}$$

(b) Based on the pressure-enthalpy diagram for methane,

$$h_1(60^\circ\text{F}, 100 \text{ psia}) \approx 398 \text{ Btu lb}_m^{-1}$$

$$h_2' (P=400 \text{ psia}, S_2 = S_1) \approx 504 \text{ Btu lb}_m^{-1}$$

Work requirement for reversible process is

$$W_s = h_2' - h_1 = 106 \text{ Btu lb}_m^{-1}$$

For a compressor with 80% efficiency, the actual work requirement is

$$W_s = 106 \text{ Btu lb}_m^{-1} / 0.8 = 132.5 \text{ Btu lb}_m^{-1}$$

The enthalpy of the exit methane under the actual condition is

$$h_2 = h_1 + 132.5 = 530.5 \text{ Btu lb}_m^{-1}$$

which corresponds to

$$T_2 \approx 295^\circ\text{F}$$

The power requirement is given by

$$dW_s/dt = (132.5 \text{ Btu lb}_m^{-1})(10 \text{ lb}_m \text{ s}^{-1}) = 1325 \text{ Btu s}^{-1}$$

$T_i/\text{K} =$	288.7			
i	$T^{(i)}/\text{K}$	$f(T^{(i)})$	$f'(T^{(i)})$	$T^{(i+1)}/\text{K}$
0	400	0.09647	0.0133	392.77
1	392.77	0.00591	0.0134	392.32
2	392.32	0.00037	0.0134	392.30
3	392.30	0.00002	0.0134	392.30

The work requirement for the reversible process is calculated for this exit temperature:

$$W_s = \Delta H = (8.314 \text{ J mol}^{-1})[A(T_2 - T_1) + (B/2)(T_2^2 - T_1^2) + (C/3)(T_2^3 - T_1^3)] = 3911.4 \text{ J mol}^{-1}$$

For the actual process, where the compressor efficiency is 0.80, the work requirement is given by

$$(W_s)_{\text{actual}} = W_s / 0.8 = 4889.2 \text{ J mol}^{-1}$$

The power requirement is therefore

$$dW_s/dt = (4889.2 \text{ J mol}^{-1})(1 \text{ mol} / 16.05 \text{ g})(10 \text{ lb}_m \text{ s}^{-1})(453.6 \text{ g/lb}_m)(0.947831 \times 10^{-3} \text{ Btu/J}) = 1310 \text{ Btu s}^{-1}$$

This means the exit temperature is necessarily higher than 392.3 K and can be found by solving the equation

$$\Delta H = (8.314 \text{ J mol}^{-1})[A(T_2 - T_1) + (B/2)(T_2^2 - T_1^2) + (C/3)(T_2^3 - T_1^3)] = 4889.2 \text{ J mol}^{-1}$$

Again we may use Newton's method to solve the above equation for T_2 :

$$T^{(i+1)} = T^{(i)} - g/g'$$

where

$$g = (8.314)[A(T - T_1) + (B/2)(T^2 - T_1^2) + (C/3)(T^3 - T_1^3)] - 4889.2$$

$$g' = (8.314)(A + B T + C T^2)$$

T_2 is found to be $415.74 \text{ K} = 288.7^\circ\text{F}$

$\Delta H / (\text{J mol}^{-1}) =$	4889.2			
i	$T^{(i)}/\text{K}$	$g(T^{(i)})$	$g'(T^{(i)})$	$T^{(i+1)}/\text{K}$
0	500	3786.4750	47.4023	420.12
1	420.12	186.5287	42.6938	415.75
2	415.75	0.5768	42.4296	415.74
3	415.74	0.0000	42.4288	415.74

The mole fraction of benzene in the bubblepoint vapor is

$$y_1 = x_1 P_1^{\text{sat}}/P = 0.8921.$$

Overall material balance:

$$L + V = 70 \text{ mol} + 30 \text{ mol} = 100 \text{ mol}$$

Material balance on benzene:

$$0.6 L + 0.8921 V = 70 \text{ mol}$$

Upon solving these simultaneous equations, one obtains

$$V = (70 \text{ mol} - 100 \text{ mol} \times 0.6)/(0.8921 - 0.6) = 34.235 \text{ mol}$$

$$L = 100 \text{ mol} - 34.235 \text{ mol} = 65.765 \text{ mol}$$

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Solution:

The bubblepoint temperature iteration formula based on the procedure given in the textbook for Raoult's law binary system has the form

$$t^{(i+1)} = -C_2 + \frac{B_2}{A_2 - \ln \left[\frac{P}{x_1 \exp \left(A_1 - \frac{B_1}{t^{(i)} + C_1} - A_2 + \frac{B_2}{t^{(i)} + C_2} \right) + (1 - x_1)} \right]}$$

where the Antoine equation coefficients refer to

$$\ln (P_i^{\text{sat}}/\text{kPa}) = A_i - B_i/[t(^{\circ}\text{C}) + C_i]$$

An alternative (and more powerful) method to find the bubblepoint temperature is to use Newton's method:

$$t^{(i+1)} = t^{(i)} - F/F'$$

where

$$F = x_1 P_1^{\text{sat}} + x_2 P_2^{\text{sat}} - P$$

$$F' = dF/dT = x_1 P_1^{\text{sat}} B_1/(t^{(i)} + C_1)^2 + x_2 P_2^{\text{sat}} B_2/(t^{(i)} + C_2)^2$$

and the vapor pressures are evaluated at the temperature $t^{(i)}$.

As shown in the accompanying Excel file, for $P = 100$ (kPa), and $x_1 = 0.6$, the converged value of t is **93.0 °C**